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PARADOXES IN EQUILIBRIUM TRAFFIC ASSIGNMENT AND THEIR IMPLICATIONS FOR OPTIMAL PLANNING STRATEGIES

by

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ABSTRACT

In a recent paper it was shown using a simple network model that the total travel cost in a user equilibrium assignment may decrease if some input flows are increased. It is shown here using the City of Winnipeg network data that this phenomenon may occur in 'real life'. The significance of these results for the problem of determining optimal planning strategies and network modelling are discussed.

INTRODUCTION

A recent study by Fisk (1979) into the sensitivity of the user equilibrium assignment solution (see, for example, Dafermos, 1971; Wardrop, 1952) to changes in input flows revealed that for certain network configurations the model exhibits properties which are counter-intuitive; namely that an increase in input flows can lead to a decrease in total travel costs. These findings are extended here to show that this phenomenon may occur in 'real life' using calibrated network data for the City of Winnipeg. The significance of these results for the problem of determining optimal planning strategies is discussed. Also we examine some of the implications of these results for network design modelling where the simplifying assumption that the traffic assignment is system optimized is often made.

SOME PARADOXICAL PROPERTIES OF THE USER EQUILIBRIUM ASSIGNMENT

Braess (see Murchland, 1970) has shown that for certain network configurations it is possible to reduce origin/destination (O/D) and total travel costs at equilibrium if a link is removed from the network. More recently it has been shown by Fisk (1979) that for the network in figure 1 with certain input flows and link cost functions, the user optimized (UO) total travel cost may decrease if the flow from a to b is incresed. Analogous counter-intuitive properties appear if the free flow travel time on link (ab) is increased or the capacity decreased.

For example suppose the initial cost functions on links (ab), (bc) and (ac) are respectively $f_{ab} + e$, f_{bc} and $f_{ac} + d$ where f designates the flow and d and e are constants. If x and y are respectively the flows on

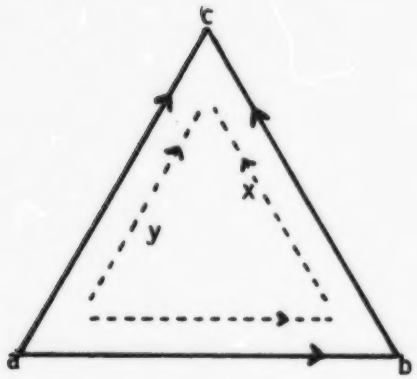

Figure 1

on paths (abc) and (ac) between a and c the network equilibrium conditions with travel demands g_{ab}, g_{bc}, and g_{ac} imply:

$$g_{ab} + g_{bc} + 2x + e = y + d$$

$$x + y = g_{ac}$$

$$0 \leq x \leq g_{ac}$$

Solving for x and y we find that

$$x = \frac{1}{3}[g_{ac} - (g_{ab} + g_{bc}) + d - e]$$

$$y = \frac{1}{3}[2g_{ac} + g_{ab} + g_{bc} - d + e]$$

and the total UO cost will be given by

$$C = u_{ab}g_{ab} + u_{ac}g_{ac} + u_{bc}g_{bc}$$

where u designates the equilibrium O/D travel cost.

If we set $d = 120$, $g_{ab} = 5$, $g_{bc} = 10$ and $g_{ac} = 90$ and increase the free flow travel cost e on link ab from 10 to 35, C will decrease from 10183,3 to 9600. On the other hand suppose the delay function for link ab is replaced by $f_{ab}/c + 35$ with $c \geq 1$. A change in capacity can be represented in this model by a change in c. It can be shown from the equilibrium conditions that $\partial C/\partial c > 0$; for $c = 1$, $C = 10183,3$ and $\lim_{c \to \infty} C = 10287,5$.

These latter two properties of the UO solution are similar to those reported by Smith (1978) who showed that for certain network configurations, total travel costs could be decreased by increasing travel costs locally at a congested intersection.

TESTS ON THE WINNIPEG NETWORK

Since detailed network data for the City of Winnipeg were available, we were able to perform tests to determine if examples of the phenomena described above could be found on a localized section of the network. The link delay curves have the BPR form; details concerning their calibration can be found in Florian *et al.* (1978). The computer code for the equilibrium assignment was originally written by Nguyen and James (1975).

Analysis showed that these phenomena are likely to occur when section (bc) of path (abc) is heavily contested and path (ac) uncongested but longer. Two examples, shown in figures 2 and 3, where the Winnipeg network has these characteristics locally were found.

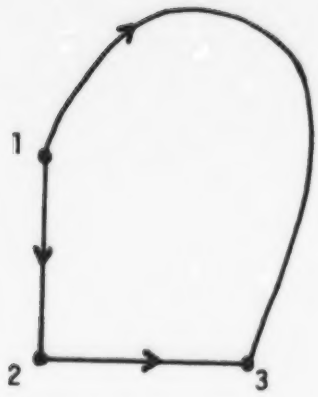

Figure 2

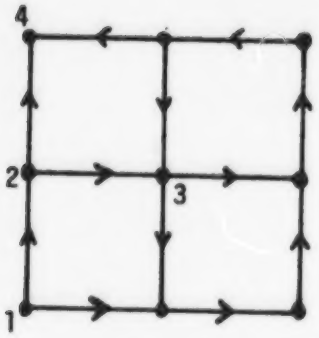

Figure 3

In figure 2 the longer route between nodes 1 and 3 has the nature of a bypass. If the initial input flows are g_{12}=50, g_{13}=200, g_{23}=3 650, the rest being zero, the initial global cost is 93 627,5. If g_{12} is increased to 250 the total cost (measured as hours of travel) decreases to 98 614,8.

In figure 3 two routes of different length between nodes 1 and 3 exist because of the structure of the one-way street system. If the initial demands are g_{13}=100, g_{14}=170, g_{23}=3 000, the total cost of travel on this network is 135,87 hours. When g_{14} is increased to 260 the total cost decreases to 135,34.

Additional tests were performed to examine the sensitivity of these networks to changes in link capacities. If the capacity of link (12) in figure 2 is reduced, trips are re-routed from the congested route (123) to the by-pass route (13). The variation of the total cost as a function of g_{12} is shown in figure 4 for the 8 delay curves given in figure 5. It is apparent that for certain ranges of g_{12} the total cost is smallest for

the delay function representing the least capacity. In each case the cost rises rapidly when all flow between nodes 1 and 3 is assigned to path (13). Furthermore it was found that when the value of g_{12} is such that both paths are candidates for flow, the total travel cost could be reduced further if a left turn at node 2 is prohibited.

Similar results were found for the network in figure 3. Figure 6 shows the variation of the total cost as a function of g_{14} in cases where link (12) has 1 and 2 lanes. For values of g_{14} where 2 paths between nodes 1 and 3 are used, the cost is smaller when link (12) has 1 lane.

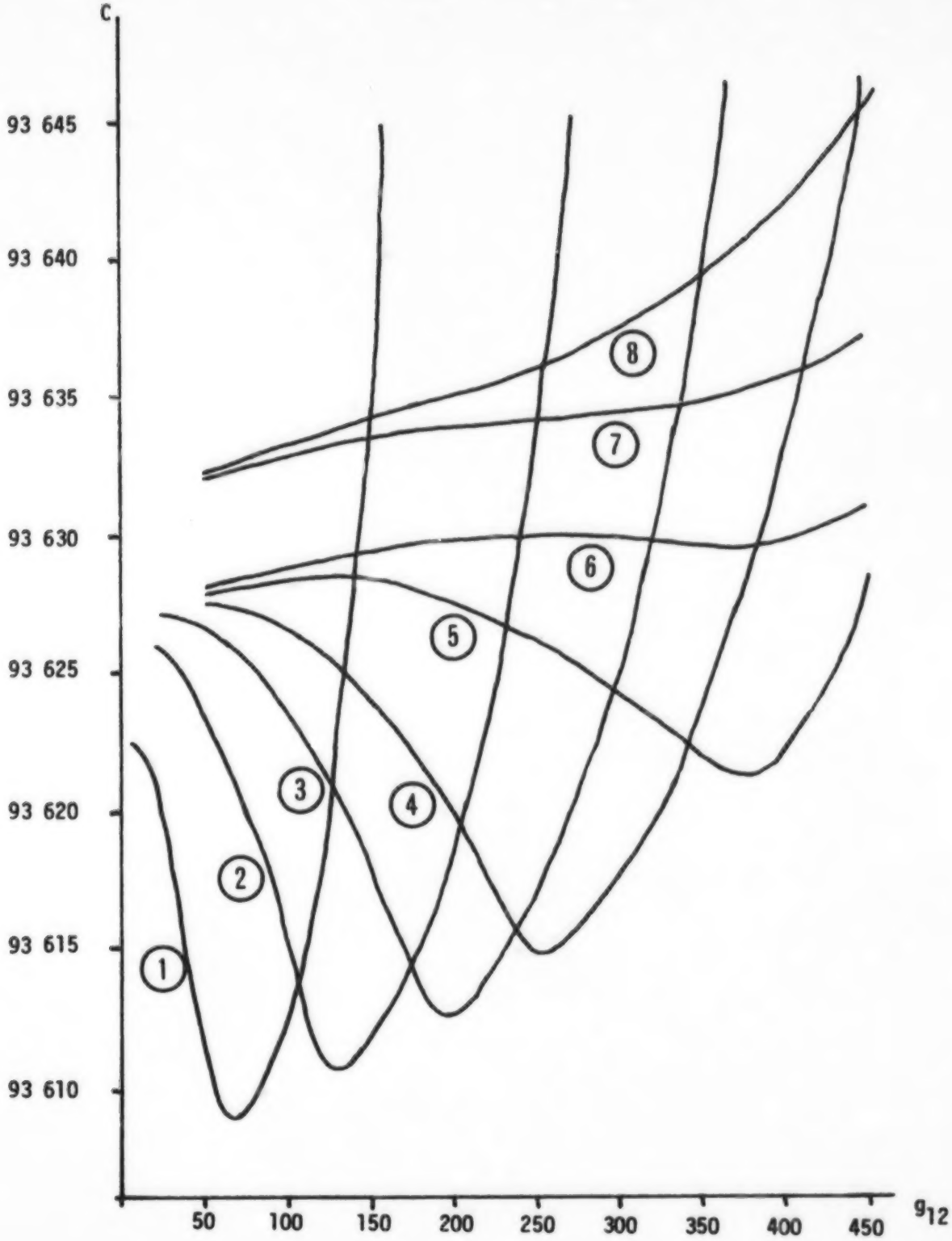

Figure 4: Variation of total cost as a function of g_{12} for 8 different link cost curves

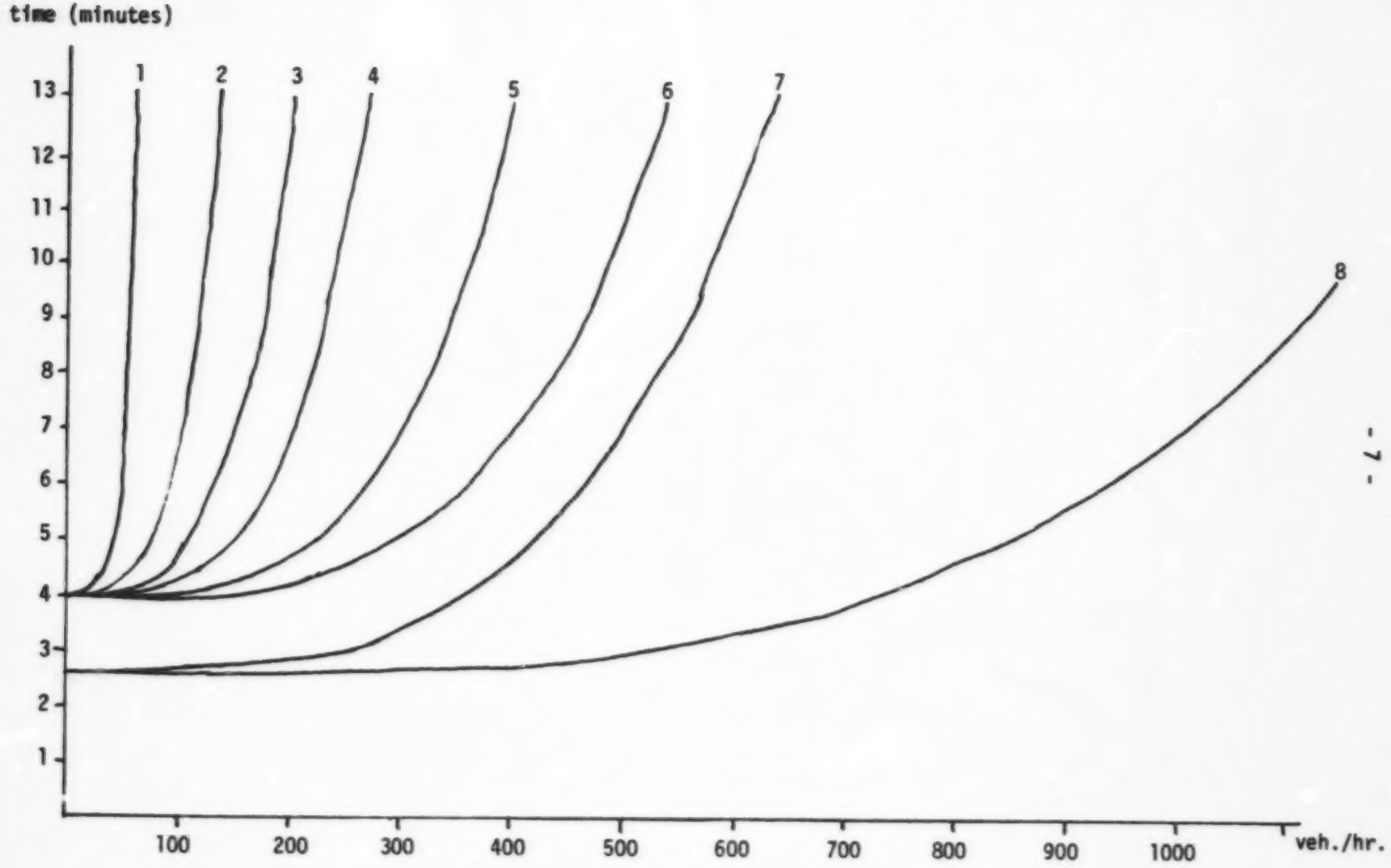

Figure 5: Eight cost functions for link (12) used to produce curves of figure 5.

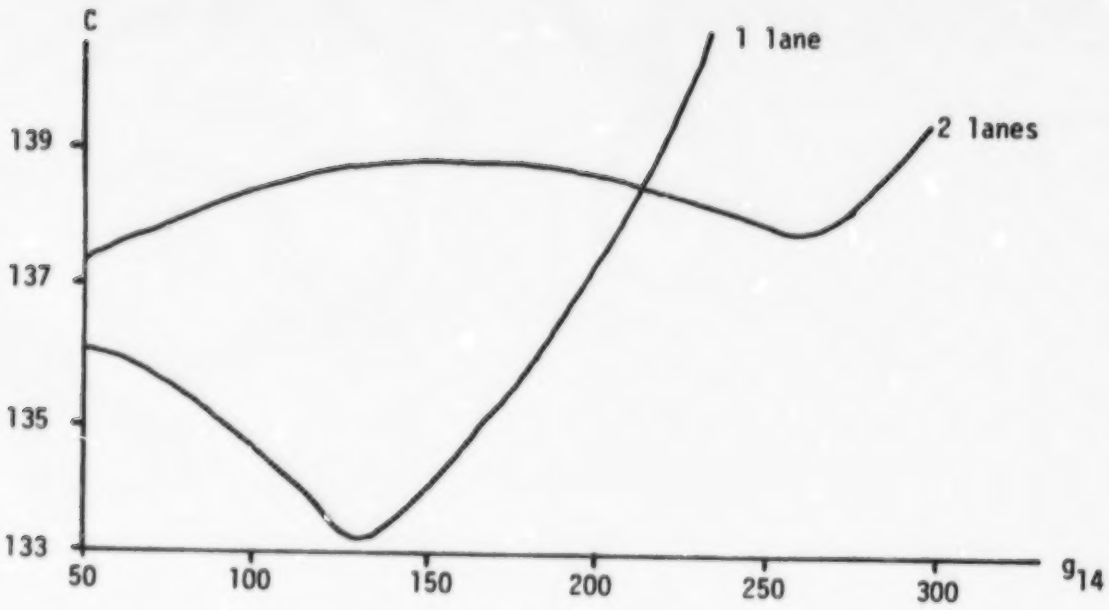

Figure 6: Variation of total cost as a function of g_{14} for network in figure 4

IMPLICATIONS FOR TRANSPORTATION PLANNING

Generally the significance of these results for transportation planning is that if the user equilibrium model is a good approximation of reality (and this is generally accepted to be the case), some planning strategies may exist for a given network which yield improvements both for traffic and the environment. Extending the results of the previous paragraphs, the potential for such improvements appears to arise when there is at least one heavily congested route (r_1) between an origin and a destination (congested with traffic travelling between other O/D pairs), and at least one route (r_2) that is longer but uncongested. Diversion of traffic from r_2 to r_1 by increasing the travel cost on some uncongested links of r_2 by any of the methods discussed above may result in a reduction of total travel costs. As an illustration we examine one scenario and discuss possible remedies for improving the network performance.

Suppose that route (abc) is a freeway with entry and exit ramps at several points including b and that section (ab) is lightly congested and section (bc) heavily congested. Route (ac) is an alternative route with a higher free flow travel time possibly due to speed restrictions, presence of signals, etc. In fact in practice (ac) would probably be the route the freeway was designed to replace.

Option 1. Increase the flow of traffic between a and b by attracting additional vehicles to b. This could be realized in the long term by appropriate land-use changes in the vacinity of b. In the short run it could be realized by discouraging traffic from using alternative routes between a and b.

Option 2. Reduce the capacity (e.g. number of lanes) on section ab of the freeway by the introduction of exclusive bus or carpool lanes, etc.

Option 3. Reduce the maximum free flow speed on section ab. This could have the additional side benefit of increasing the energy efficiency of vehicles using ab.

Implementation of such policies would require considerable faith in the ability of the equilibrium model to reproduce the actual traffic patterns. However this is true of any policy implemented on the basis of model predictions.

COMPARISON BETWEEN THE SO AND UO ASSIGNMENTS

Clearly the potential for improvement of the performance of a network arises because of the difference in the UO and system optimal (SO) solutions. We now attempt to explain the above phenomena by comparing the results obtained from these two assignment methods.

A comparison between the SO and UO assignments can be readily made if the SO problem is reformulated as an equivalent UO problem. (See, for example, Dafermos, 1971). It can be shown from the Kuhn-Tucker conditions for the two problems that the SO problem is equivalent to the UO problem with transformed link costs: $s'_a = s_a + f_a \frac{\partial s_a}{\partial f_a}$ where s_a is the cost function for link a. For uncongested networks $\frac{\partial s_a}{\partial f_a}(\hat{f}_a) \sim 0$ where $\hat{f}$ designates the SO solution; consequently the SO solution will closely resemble the UO solution. For higher levels of congestion the SO method will tend to

allocate more flow to uncongested routes; for a given level of travel demand this bias will be more apparent the larger the gradient of the link cost curves. (The SO and UO results are identical if $\frac{\partial s_a}{\partial f_a} = 0$).

Some of these properties are confirmed by the experimental results given in LeBlanc *et al.* (1976). The SO pattern generally has more flow on expressway links and less flow on congested arterial links than the UO pattern. This bias is also readily apparent from the observation that the average travel speed is higher for the SO assignment.

EXPLANATION OF PARADOXES

As is well known Braess's paradox is a direct consequence of the difference between the UO and SO assignment solutions. Prohibiting the traffic on the diagonal link in his example by turning restrictions for example, forces the UO solution to become the SO solution. This argument can be extended to explain the apparent paradoxical properties of the user equilibrium assignment for the network in Fig. 1. An increase in demand between a and b forces some traffic from the congested route (abc) to the uncongested route (ac) bringing the UO solution closer to the SO solution. The resulting decrease in total cost more than offsets the increase in cost to traffic using (ab). A similar explanation applies to the other examples discussed.

IMPLICATION FOR THE NETWORK DESIGN PROBLEM

The results of the preceding sections raise the question of the acceptability of the simplifying assumption often made in the network design problem that the assignment is system optimized. For example if the benefits of options 1-3 in the scenarios discussed above are tested using a SO assignment, a before and after comparison will show an increase in total travel costs. More generally the bias in a SO assignment of allocating more traffic to uncongested routes will bias to some extent the measure of benefits of any network changes.

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